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The reflection and transmission group delay times in an asymmetric single quantum barrier

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Abstract. The reflection and transmission group delay times are systematically investigated in an asymmetric single quantum barrier. It is reported that the reflection times in both evanescent and propagating cases can be either negative or positive, depending on the relative height of the potential energies on the two sides of the barrier. In the evanescent case where the energy of incident particles is less than the height of the barrier, the reflection and transmission times in the opaque limit are both independent of the barrier's thickness, showing superluminality. On the other hand, in the propagating case where the energy of incident particles is larger than the height of the barrier, the reflection and transmission times as the periodical function of the barrier's thickness can be greatly enhanced by the transmission resonance. It is also shown that the transmission time and the reflection times for the two propagation directions in the same asymmetric configuration satisfy the reciprocal relation, as consequence of time reversal invariance in quantum mechanics. These phenomena may lead to novel applications in electronic devices.

PACS. 03.65.Xp Tunneling, traversal time, quantum Zeno dynamics – 73.23.-b Electronic transport in mesoscopic systems – 03.65.-w Quantum mechanics

1 Introduction

The tunneling time of quantum particles through single or multiple quantum barriers has drawn considerable attention [1–5] for last decades with the advent of techniques for the fabrication of semiconductor tunneling devices, such as single-electron tunneling transistors [6], resonant tunneling diodes [7], quantum cascade lasers [8], and resonant photodetectors [9]. Theoretical investigations [10–17] and experimental researches [18–27] have been attempted to determine a physical tunneling time. However, there is still lack of consensus about the existence of a simple expression for this time, due to the fact that there is no Hermitian operator associated with it in quantum mechanics [4]. In recent years, the prospect of hight-speed nanoscale electric devices, based on the tunneling process, has brought new urgency to the analysis of the tunneling time, as it is directly related to the maximum attainable speed of such devices [5]. There were several plausible tunneling times proposed to this problem in terms of different operational definitions and physical interpretations. Different approaches lead to the expressions for "tunneling time", which imply the possibility of superluminal tunneling velocities in certain cases [1,3].

Among various time scales, the group delay time (also referred to as the phase time in the literature [1]), which describes the motion of a wave packet peak [28], has well-known superluminality [3–5]. It was found that the group delay time for quantum particles tunneling through a quantum barrier become independent of the thickness of the barrier in the opaque limit. This phenomenon is often termed as the "Hartman effect" [29]. With the experimental verifications of the Hartman effect, the superluminal group delay times have been directly measured in a series of famous microwave or optical analogy experiments [20–23]. It is important to note that these observations don't violate "Einstein causality". There is no causal relationship between the peaks of the incident and transmitted packets [10]. In addition, the group delay time has been an interesting quantity in quantum coherent electron transport. The group delay statistics is intimately connected to the dynamic admittance and other properties of microstructure [30]. The group delay time is also related to the density of state [31,32]. Recently, the concept of negative group delay has been extended to microelectronics [33], and the Hartman effect has been further

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investigated in the quantum ring geometry in present of Aharanov-Bohm flux and the quantum networks [34].

In our previous works, we have elaborated that the quantum particles traveling through a quantum well can be advanced [35], so that the group delay in transmission can be negative [36]. This counterintuitive phenomenon has been demonstrated in a microwave analogy experiment [37]. In reflection, the negative group delay time has also been reported in asymmetric double-barrier quantum wells [38,39] and their optical analogies [40,5]. It is worthwhile to point out that the negative group delay times, which were previously discovered in both reflection and transmission, are always related to the quantum-well structures. As a matter of fact, it will be shown that the negative group delay times can also take place in a single potential barrier of asymmetric configuration.

As inspired by the experimental observation of negative differential resistance [41] and above-barrier quasi-bound state [42] in single quantum barrier, the tunneling times in different kinds of single barrier structures have renewed much attention [43,44]. With some exceptions [45–47], most theoretical works on tunneling times were merely discussed in a spatially symmetric barrier. As described in the literature [47], the tunneling times in the asymmetric barrier have much notable difference from those in the symmetric one. Recently, Li and Spieker [48] have investigated that the resonance-enhanced group delay times can be negative as well as positive, when the quantum particles are scattered by an asymmetric single potential barrier, the height of which is less than the energy of incident particles. Meanwhile, the theoretical predictions have been tested in a microwave experiment. However, the superluminal and even negative group delay time in an asymmetric single potential barrier have rarely been considered in the evanescent case where the energy of incident particles is less than the height of the barrier. It remains an open question.

The main purpose of this paper is to investigate the transmission and reflection group delay times for quantum particles through an asymmetric single potential barrier systematically. Depending on the incidence energy of quantum particles, the traversal of the particles through a potential barrier can be divided into two cases, the propagating and evanescent cases. It is reported in present paper that the reflection times can be negative as well as positive in both evanescent and propagating cases. The negative reflection times are closely related to the relative height of the potential energies on the two sides of the potential barrier, and always correspond to a maximum transmission coefficient that is larger than unity. In the evanesce case, it is shown that the reflection and transmission times are both independent of the barrier's thickness in the opaque limit, showing superluminality. On the other hand, the group delay times as the periodical function of the barrier's thickness can be greatly enhanced by transmission resonance in the propagating case. As consequence of the time-reversal symmetry in quantum mechanics, it is also shown that the reflection times for the two propagation directions are quite different in the same asymmetric configuration, while their average time equals to the unique transmission time.

This paper is arranged as follows. We start in Section 2 with the derivation of the group delay times in the evanescent case, and establish their reciprocal relation in the same asymmetric configuration. Thereafter, in Section 3, the superluminal and even negative properties of the group delay times are discussed in detail. In Section 4, we turn to the resonance-enhanced group delay times in the propagating case and present the relationship between the resonant group delay times and quasi-bound states lifetime in the barrier region. Finally, we summarize the paper in Section 5.

2 Evanescent case

Consider a beam of particles propagating towards an asymmetric rectangular potential barrier of the height, V_0, extending from 0 to a, as shown in Figure 1, where the height of the potential energies on the left and right sides of the barrier are V_1 and V_2, respectively. It is reasonably assumed that $V_1, V_2 < V_0$. First of all, what we consider is such an evanescent case, in which the energy of incident particles E is less than the height of the barrier, $V_1, V_2 < E < V_0$. When a beam of particles comes from the left, let be $\psi_{in}(x) = A\exp(ik_1x)$ the Fourier component of the incident wave packet, where $k_1 = [2\mu(E-V_1)]^{1/2}/\hbar$, μ is the mass of incident particles. Denoting, respectively, by $B\exp(-ik_1x)$ and $F\exp[ik_2(x-a)]$ the corresponding Fourier components of the reflected and transmitted wave packets, then the Schrödinger equation and boundary conditions at $x=0$ and $x=a$ give $r \equiv B/A = (g_2/g_1)\exp[i(\phi_1-\phi_2)]$and $t \equiv F/A = (1/g_1)\exp(i\phi_1)$, where $k_2 = [2\mu(E-V_2)]^{1/2}/\hbar$, $\kappa = [2\mu(V_0-E)]^{1/2}/\hbar$, non-negative number g_1 and real number ϕ_1 are defined by a complex number as follows,

$$g_1\exp(i\phi_1) = \frac{1}{2}\left(1+\frac{k_2}{k_1}\right)\cosh\kappa a + \frac{i}{2}\left(\frac{k_2}{\kappa}-\frac{\kappa}{k_1}\right)\sinh\kappa a, \tag{1}$$

and non-negative number g_2 and real number ϕ_2 are defined similarly by another complex number as follows,

$$g_2\exp(i\phi_2) = \frac{1}{2}\left(1-\frac{k_2}{k_1}\right)\cosh\kappa a + \frac{i}{2}\left(\frac{k_2}{\kappa}+\frac{\kappa}{k_1}\right)\sinh\kappa a. \tag{2}$$

According to the definition (1), we have

$$\tan\phi_1 = \frac{1/\kappa - \kappa/k_1k_2}{1/k_2+1/k_1}\tanh\kappa a \tag{3}$$

which shows that the phase ϕ_1 of the transmission coefficient is symmetric between k_1 and k_2. Likewise, we can obtain from the definition (2) that,

$$\tan\phi_2 = \frac{1/\kappa + \kappa/k_1k_2}{1/k_2-1/k_1}\tanh\kappa a, \tag{4}$$

which shows that ϕ_2 will change its sign by exchanging k_1 and k_2. This property of asymmetry will have important effect on the group delay time in reflection. We can

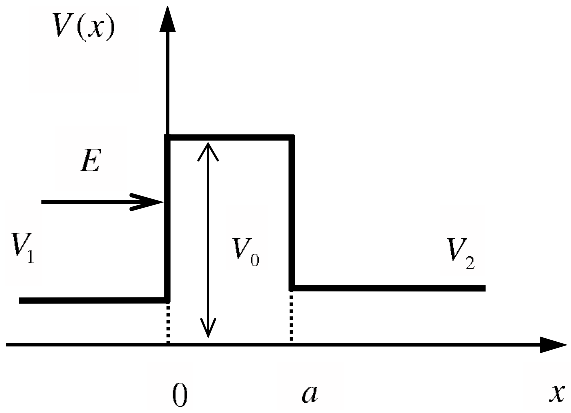

Fig. 1. Schematic diagram of the particles propagating through an asymmetric single barrier from the left in the case where the energy of incident particles is less than the height of the barrier.

also see from equations (3) and (4) that ϕ_1 and ϕ_2 can be exchanged from one to another by changing the sign of k_1. This symmetry between ϕ_1 and ϕ_2 will simplify our calculation for the group delay time.

It is noted that $\phi_1 - \phi_2$ is the total phase shift of the reflected wave packet, and ϕ_1 itself is the total phase shift of the transmitted wave packet, rather than defined relatively to a free particle in the same configuration. The group delay time in reflection, according to stationary-phase theory [14,28], is $\tau_r = \hbar d(\phi_1 - \phi_2)/dE$. The group delay time in transmission, which is defined as the derivative of the phase shift ϕ_1 with respect to particle's energy E [14,47], is given by,

$$\tau_t = \hbar \frac{d\phi_1}{dE} = \frac{\tau_{sc}}{4g_1^2}\left(1+\frac{k_2}{k_1}\right) \times \left[\frac{\kappa}{k_1} - \frac{k_2}{\kappa} + \left(1+\frac{\kappa^2}{k_1^2}\right)\left(\frac{k_2}{\kappa}+\frac{\kappa}{k_2}\right)\frac{\sinh 2\kappa a}{2\kappa a}\right], \quad (5)$$

where $\tau_{sc} = \mu a/\hbar\kappa$ is the "semiclassical time", also referred as the Büttiker-Landauer time [10], taken for particles to travel through the barrier region. Substituting ϕ_1 defined in equation (3) and noticing equation (5), we finally obtain $\tau_r = \tau_t - \tau_0$ for the group delay time of the reflected wave packet, where

$$\tau_0 = \hbar \frac{d\phi_2}{dE} = -\frac{\tau_{sc}}{4g_2^2}\left(1-\frac{k_2}{k_1}\right) \times \left[\frac{\kappa}{k_1} + \frac{k_2}{\kappa} - \left(1+\frac{\kappa^2}{k_1^2}\right)\left(\frac{k_2}{\kappa}+\frac{\kappa}{k_2}\right)\frac{\sinh 2\kappa a}{2\kappa a}\right]. \quad (6)$$

As is apparent, the transmission time (5) shows an important symmetry with respect to k_1 and k_2, resulting from the property of ϕ_1. On the contrary, τ_0 is of asymmetry between k_1 and k_2, because of the aforementioned property of ϕ_2. That is to say, the reflection times $\tau_r = \tau_t - \tau_0$ is closely related to the relative height of the potential energies on the two sides of the potential barrier. In other words, the reflection times are dependent of the directions of incident particles upon the barrier in the same asymmetric structure.

In order to establish the universal relationship between the transmission and reflection group delay times in the same asymmetric configuration, we introduce transmission time τ_t and two distinct reflection times, τ_r^+ and τ_r^-, where τ_r^+ and τ_r^- stand for the reflection times for the particles coming from the left and right side of the barrier, respectively. Hence, the transmission time τ_t and the reflection times τ_r^+, τ_r^- are not independent but satisfy the relation, $\tau_t = (\tau_r^+ + \tau_r^-)/2$ [14,40]. This reciprocal relation can be obtained from the unitary relation of one-dimensional scattering matrix [4]. In fact, it is a general result of the time-reversal symmetry that reflects the microscopic reversibility of quantum mechanics itself. In a symmetric configuration, $\tau_0 = 0$ will vanish for $k_1 = k_2$, so that $\tau_t = \tau_r^+ = \tau_r^-$ [10]. This indicates that the properties of the reflection times do result from the asymmetry of the barrier, reflected in the phase difference between the overall reflection coefficients for the two tunneling directions.

3 Superluminal and negative properties

In this section, we are concerned with the superluminal and negative properties of the group delay times in the evanescent case. As obtained in previous section, the reflection time $\tau_r = \tau_t - \tau_0$ is quite different from the transmission time τ_t in an asymmetric single potential barrier. Noting that $g_2^2 < g_1^2$, the magnitude of τ_0 can be larger than τ_t. Therefore, the reflection time $\tau_r = \tau_t - \tau_0$ can be negative in evanescent case. To simplify the discussions, we consider the negative property of the reflection time in the case of $k_1 \sim k_2$ and $\kappa \ll k_1$. When $k_1 \sim k_2$ and $\kappa \ll k_2$, the reflection time $\tau_r = \tau_t - \tau_0$ can be approximately simplified as

$$\tau_r \approx \frac{\tau_{sc}}{8g_1^2 g_2^2}\left[\left(\frac{k_2}{\kappa}+\frac{\kappa}{k_2}\right)\frac{\sinh 2\kappa a}{2\kappa a} - \frac{k_2}{\kappa}\right] \times \left[\left(1+\frac{k_2^2}{\kappa^2}\right)\sinh^2 \kappa a - \left(1-\frac{k_2^2}{k_1^2}\right)\right]. \quad (7)$$

It is seen from equation (7) that the factor

$$\left[\left(1+\frac{k_2^2}{\kappa^2}\right)\sinh^2 \kappa a - \left(1-\frac{k_2^2}{k_1^2}\right)\right],$$

can make the reflection time τ_r negative. Mathematically, the above factor is always positive when $k_2 > k_1$ ($V_1 > V_2$). This means that the reflection time always shows positivity when $k_2 > k_1$ ($V_1 > V_2$). On the other hand, this factor can be either negative or positive when $k_2 < k_1$ ($V_2 > V_1$). Since the function $\sinh \kappa a$ increases rapidly with increasing the barrier's thickness, the negative value will become positive beyond a critical barrier's thickness. That is to say, the negative reflection time occurs only for a small barrier's thickness. As a result, the condition $k_2 < k_1$ ($V_2 > V_1$) is not sufficient but necessary for the reflection time to be negative. Next, the features of the group delay times in reflection and transmission are further discussed for the opaque and transparent barriers, respectively.

(a) Opaque limit: When $a \gg 1/\kappa$ with k_1, k_2 and κ remaining finite, the transmission time in (5) has the following form,

$$\lim_{\kappa a \to \infty} \tau_t = \frac{\mu}{\hbar\kappa}\left(\frac{1}{k_1} + \frac{1}{k_2}\right),$$

which is independent of the thickness a of the barrier. So beyond a critical value of barrier's thickness, τ_t will be smaller than the barrier's thickness divided by the vacuum speed of light, showing superluminality. This is the so-called Hartman effect [29,47]. In this limit, τ_0 tends to

$$\lim_{\kappa a \to \infty} \tau_0 = \frac{\mu}{\hbar\kappa}\left(\frac{1}{k_2} - \frac{1}{k_1}\right),$$

so that the reflection group delay time saturates in opaque limit to a constant value,

$$\lim_{\kappa a \to \infty} \tau_r = \frac{2\mu}{\hbar k_1 \kappa},$$

where κ is the decaying constant of the wave inside the barrier region. It is indicated that the reflection time in opaque limit also becomes independent of the barrier's thickness, which corresponds to the "Hartman effect" in reflection. Since the penetration depth $1/\kappa$ is about order of the wavelength $2\pi/k_1$, τ_r is about the order of the period of the incident wave, $h/(E - V_1)$. This result is in agreement with the universal time, which approximately equals to the reciprocal of the carrier frequency or of the wave packet energy divided by the Planck constant h [5]. When removing the asymmetry of the single barrier, $k_1 = k_2$, the reflection and transmission times take the same form, $2\mu/\hbar k_1 \kappa$, as pointed out in reference [10].

(b) Transparent limit: When $\kappa a \to 0$, for the barrier to be transparent, it is necessary that the thickness of the barrier should be much smaller than the reciprocal of κ, i.e. $a \ll 1/\kappa$. It is meant by this limit that the transmission coefficient for the particles tunneling through an asymmetric potential barrier approximately reaches maximum, thus the barrier shows the transparency. In this limit, the group delay time in transmission takes the following form,

$$\lim_{\kappa a \to 0} \tau_t = \frac{\mu a}{\hbar k_1 k_2} \frac{k_1^2 + k_2^2 + k_1 k_2 + \kappa^2}{k_1 + k_2},$$

and the reflection time $\tau_r = \tau_t - \tau_0$ can be expressed by,

$$\lim_{\kappa a \to 0} \tau_r = \frac{2\mu a}{\hbar k_2} \frac{k_1^2 + \kappa^2}{k_2^2 - k_1^2},$$

which can be negative as well as positive. When $k_1 > k_2$ ($V_1 < V_2$), τ_r is negative. On the other hand, when $k_1 < k_2$ ($V_1 > V_2$), τ_r is positive. These results are consistent with those previously obtained under the condition $k_1 \sim k_2$ and $\kappa \ll k_1$. More interestingly, when the barrier's thickness is much smaller than $1/\kappa$, the transmission coefficient $T = 1/g_1^2$ reaches the maximum value as follows,

$$T_{max} = \frac{4}{(1 + k_2/k_1)^2}.$$

It is clear that the transmission coefficient T_{max} can be larger than unity when $k_1 > k_2$. This indicates that the negative reflection time corresponds to a maximum transmission coefficient that is larger than unity. The fact that transmission coefficient can be larger than unity doesn't violate the law of probability current conservation. The transmission coefficient T is the ratio of the amplitude, rather than the transmission probability, $(k_2/k_1)T$, defined as the ratio of the probability currents carried by the transmitted wave over the incident wave. In fact, the transmission probability $(k_2/k_1)T$ is always less than unity.

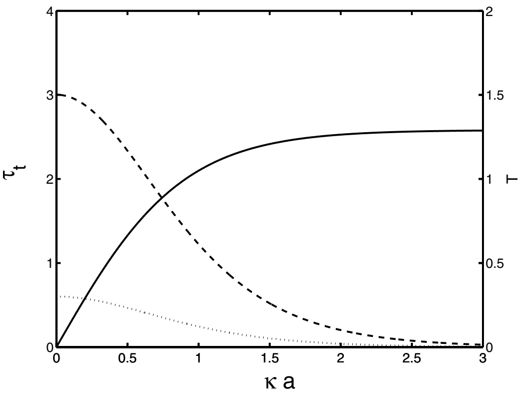

Fig. 2. The dependence of the group delay time in transmission τ_t and the transmission coefficient T on the thickness a of the barrier, where a is re-scaled to κa and the unit of $\hbar$ and m is set to unity. The dashed curve corresponds to the transmission coefficient T, where $V_0/E = 2$, $V_1/E = 0.8$, and $V_2/E = 0.9$. The dotted curve corresponds to the transmission probability T, where $V_0/E = 2$, $V_1/E = 0.9$, and $V_2/E = 0.8$. Here the transmission time is also depicted by solid curve, which is identical in such two cases as $V_1 > V_2$ and $V_1 < V_2$.

In Figure 2 is shown the dependence of the group delay time in transmission τ_t and the transmission coeffient T on the thickness a of the barrier, where a is re-scaled to κa and the unit of $\hbar$ and m is set to unity. The dotted curve corresponds to the transmission probability that is less than unity, where $V_0/E = 2$, $V_1/E = 0.9$, and $V_2/E = 0.8$. Conversely, the dashed curve corresponds to the transmission coefficient that is larger than unity, where $V_0/E = 2$, $V_1/E = 0.8$, and $V_2/E = 0.9$. As shown in Figure 2, the transmission times in two cases are always equal to the identical positive value, due to the symmetry. It is also shown that the transmission time is independent of the thickness a of the barrier in the opaque limit, which confirms the Hartman effect.

Figure 3 shows the dependence of the group delay times in reflection τ_r on the thickness a of the barrier,

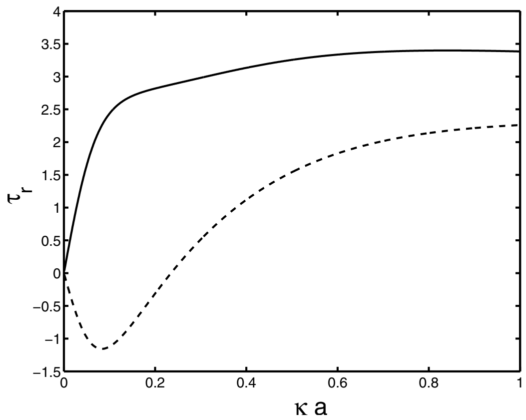

Fig. 3. The dependence of the group delay time in reflection τ_r on the thickness a of the barrier, where all the physical parameters are the same in Figure 2, a is re-scaled to κa, and the unit of $\hbar$ and m is set to unity. The solid curve corresponds to the positive reflection time when $V_1 > V_2$. The dashed curve corresponds to the negative reflection time when $V_1 < V_2$.

where all the corresponding physical parameters are the same as in Figure 2, a is re-scaled to κa and the unit of $\hbar$ and m is set to unity. The solid curve indicates the positive reflection time when $V_1 > V_2$. Nevertheless, the dashed curve indicates the negative reflection time when $V_1 < V_2$. It turns out that the reflection times have close relation to the relative height of the potential energies on the two sides of the barrier. More interestingly, it is seen from Figure 3 that the negative reflection time for $V_2 > V_1$ can be changed to a positive value when increasing the barrier's thickness a, and finally saturate to a positive constant value for a sufficiently large barrier's thickness.

From all these discussions, we draw the conclusion that the reflection and transmission times for a sufficiently large barrier always become independent of the barrier's thickness. Furthermore, the reflection time can be negative in evanescent case, and the corresponding maximum transmission probability is larger than unity. As we know, the group delay time is associated with the partial density of state (PDOS) in some approximations, which represents the contribution to the local density of states [4,32]. For an arbitrary one-dimensional scattering problem, Gasparian et al. [31] mentioned there that certain partial density of states is not positive, which would lead to negative times. In this section, the negative group delay obtained can occur only for a small thickness of the barrier and its magnitude is quite small. In what follows, we will show that the reflection and transmission group delay times in the propagating case depend periodically on barrier's thickness, thus can be greatly enhanced by transmission resonance.

4 Propagating case

In this section, we now turn to the case of propagating. As shown in Figure 4, the quantum particles are scattered by an asymmetric barrier, the height of which is

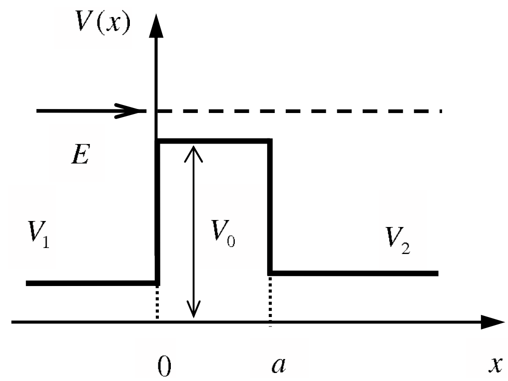

Fig. 4. Schematic diagram of the particles scattered by an asymmetric single barrier from the left in the case where the energy of incident particles is above the barrier.

less than the energy of incident particles. This is classically allowed motion. The particles in the region of barrier have the real classical moving velocity, $v_c = \hbar k/\mu$, where $k = [2\mu(E - V_0)]^{1/2}/\hbar$. By replacing κ with ik, the transmission time (5) in this case can be rewritten as [48],

$$\tau_t' = \frac{\tau_c}{4{g_1'}^2}\left(1+\frac{k_2}{k_1}\right) \times \left[\frac{k}{k_1}+\frac{k_2}{k}-\left(1-\frac{k^2}{k_1^2}\right)\left(\frac{k}{k_2}-\frac{k_2}{k}\right)\frac{\sin 2ka}{2ka}\right], \quad (8)$$

where

$${g_1'}^2 = \frac{1}{4}\left(1+\frac{k_2}{k_1}\right)^2 - \frac{1}{4}\left(1-\frac{k^2}{k_1^2}\right)\left(1-\frac{k_2^2}{k^2}\right)\sin^2 ka.$$

Then, the reflection group delay time, $\tau_r' = \tau_t' - \tau_0'$, can be calculated from

$$\tau_0' = \frac{\tau_c}{4{g_2'}^2}\left(1-\frac{k_2}{k_1}\right) \times \left[\frac{k_2}{k}-\frac{k}{k_1}+\left(1-\frac{k^2}{k_1^2}\right)\left(\frac{k_2}{k}-\frac{k}{k_2}\right)\frac{\sin 2ka}{2ka}\right], \quad (9)$$

where

$${g_2'}^2 = \frac{1}{4}\left(1-\frac{k_2}{k_1}\right)^2 - \frac{1}{4}\left(1-\frac{k^2}{k_1^2}\right)\left(1-\frac{k_2^2}{k^2}\right)\sin^2 ka.$$

As can be seen, the transmission and reflection times are closely related to the periodical occurrence of transmission resonance, and thus can be larger as well as less than the classical time, $\tau_c = a/v_c$. It is also noted that the transmission and reflection times presented here still satisfy the reciprocal relation, due to their own properties of symmetry and asymmetry [48].

To illustrate its properties clearly, we consider the reflection time in the case of $k_1 \sim k_2$ and $k \ll k_2$. Under this condition, the reflection time τ_r' will be dominated by τ_0', and its sign will be determined by the sign of τ_0', since ${g_2'}^2$

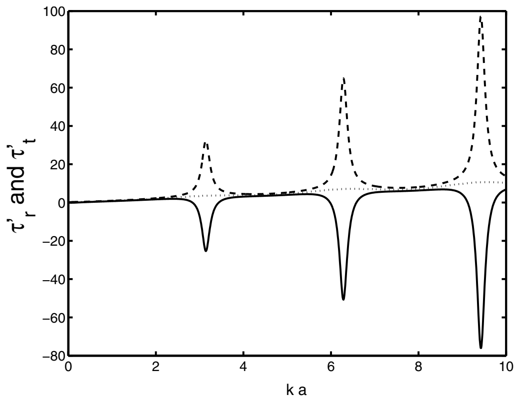

Fig. 5. The dependence of the group delay times in reflection and transmission on the thickness a of barrier, where a is re-scaled to ka and the unit of $\hbar$ and m is set to unity. The solid curve corresponds to the negative reflection time, where $V_0/E = 0.75$, $V_1/E = 0.5$, and $V_1/E = 0.8$. The dashed curve corresponds to the positive reflection time, where $V_0/E = 0.75$, $V_1/E = 0.8$, and $V_2/E = 0.5$. Here the transmission time is also depicted by dotted curve, which is identical in such two cases as $V_1 > V_2$ and $V_1 < V_2$.

is much less than ${g'_1}^2$. As a result, near the transmission resonance, τ'_r can be approximated as

$$\tau'_r \approx -\frac{\tau_c}{4{g'_2}^2}\left(1-\frac{k_2}{k_1}\right)\left(\frac{k_2}{k}-\frac{k}{k_1}\right). \tag{10}$$

It is interesting to note that the reflection time τ'_r can be either negative or positive. When $k_1 < k_2$ ($V_1 > V_2$), τ'_r is positive. On the contrary, τ'_r is negative, when $k_1 > k_2$ ($V_1 < V_2$). Compared with the reflection time (7) in evanescent case, the reflection time as periodical function of the barrier's thickness can be negative for certain large barrier's thickness.

Figure 5 shows the dependence of the group delay times in reflection and transmission on the thickness a of the barrier, where a is re-scaled to ka and the unit of $\hbar$ and m is set to unity. The solid curve corresponds to the negative reflection time, where $V_0/E = 0.75$, $V_1/E = 0.5$, and $V_1/E = 0.8$. The dashed curve corresponds to the positive reflection time, where $V_0/E = 0.75$, $V_1/E = 0.8$, and $V_2/E = 0.5$. Here the transmission time is also depicted by dotted curve. As shown in Figure 5, the transmission and reflection times are in direct connection with the periodical occurrence of transmission resonance, $ka = m\pi$ ($m = 1, 2, 3...$). Near the resonance points, the reflection times are much larger than the transmission time. It is shown that the group delay times can be greatly enhanced by one or two order, compared to those in the evanescent case. In addition, we also see that the reflection time is almost the same as the transmission one far from resonance.

Apart from the above-mentioned negativity of the reflection time, the reflection and transmission times have other interesting properties at transmission resonance deserving being pointed out. When the transmission resonance $ka = m\pi$ ($m = 1, 2, 3...$) occurs, the transmission coefficient $T' = 1/{g'_1}^2$ reaches maximum value, $T'_{max} = 4/(1 + k_2/k_1)^2$, and the transmission time reduces to

$$\tau'_{tmax} = \tau'_t|_{ka=m\pi} = \frac{k^2 + k_1k_2}{k(k_1 + k_2)}\tau_c.$$

This indicates that τ'_t reaches maximum at $ka = m\pi$, and becomes larger than the classical time τ_c. Meanwhile, the reflection probability $R' = {g'_2}^2/{g'_1}^2$ doesn't vanish, so that the reflected wave packet can be defined and the resonant reflection time is given by,

$$\tau'_r|_{ka=m\pi} = \frac{2k_1(k_2^2 - k^2)}{k(k_2^2 - k_1^2)}\tau_c.$$

We show that the reflection time can be negative when $k_1 > k_2$ ($V_1 < V_2$), while the reflection time can be positive when $k_2 > k_1$ ($V_2 < V_1$). These results are in agreement with those in the case of $k_1 \sim k_2$ and $k \ll k_2$. Obviously, the negative resonant peaks of the reflection time always correspond to a maximum transmission coefficient T' that is larger than unity. On the other hand, when $ka = (m + 1/2)\pi$, the transmission time becomes

$$\tau'_t|_{ka=(m+1/2)\pi} = \frac{k(k_1 + k_2)}{k^2 + k_1k_2}\tau_c.$$

It is implied that the transmission time far from resonance is less than the classical time τ_c and always have a positive value. Under above conditions of $k \ll k_2$ and $k_1 \sim k_2$, the reflection time at $ka = (m + 1/2)\pi$ tends to the transmission time. Furthermore, it is worth mentioning in the propagating case that the resonance condition $ka = m\pi$ for transmission through a single barrier is the same as that for the quasi-location of the states in the barrier region [42]. The resonant transmission time is of the order of the quasi-bound state lifetime in the barrier region, and the magnitude of the resonant reflection one is much larger than the quasi-bound state lifetime [45]. As a matter of fact, the negative reflection time for Breit-Wigner resonance has been already reported by M. Büttiker [38].

Because of the analogy between Schrödinger's equation in quantum mechanics and Helmholtz's equation in electromagnetism, these predictions have been observed experimentally in the so-called G-band waveguide of width 47.5 mm [48], where the asymmetric quantum barrier was obtained by reducing the inside width of the waveguide, leading to the effective widths of 40.5 mm and 30.5 mm. The resonance-enhancement of the group delay times is clearly shown, as expected by the theoretical explanation above. Incidentally, the negative group delay and Hartman effect in the evanescent case can be further demonstrated in the same experimental setup.

Finally, we consider the validity of the above theoretical results. For the validity of stationary-phase approximation, that is, for the distortion of the reflected wave packet to be negligible, the thickness of the barrier is required to be [48],

$$a \leq 2v_c w \sin^{-1}\frac{k_0|k_1 - k_2|}{[(k_1^2 - k_0^2)(k_2^2 - k_0^2)]^{1/2}}. \tag{11}$$

Within the restriction, the temporal wave packet can travel through the asymmetric barrier with negligible distortion in the case of propagation. In the tunneling regime, by contrast, the imaginary value of κa turns this periodical oscillation into a hyperbolic function. The phase varies smoothly from about $\pi/2$ at $E = V_0$ to $-\pi/2$ at $E = 0$, thus the stationary phase method doesn't break down in the tunneling process, as pointed out in the literature [14].

5 Conclusions

We have investigated systemically the group delay times in reflection and transmission for quantum particles through an asymmetric single quantum barrier. It is found that the reflection times can be negative as well as positive in both evanescent and propagating cases. The negative reflection time is closely related to the relative height of the potential energies on the two sides of the barrier, and always corresponds to that a maximum transmission coefficient is larger than unity. In the evanescent case, the reflection and transmission times for an opaque barrier are always independent of the barrier thickness, which are the so-called "Hartman effect" in reflection and transmission. When the energy of the incident particles is above the potential barrier, the reflection and transmission times as the periodical function of the barrier thickness can be greatly enhanced by transmission resonance. In addition, we also show that the transmission and reflection times satisfy the reciprocal relation in the same asymmetric configuration from time-reversal symmetry. In fact, these negative group delay times in reflection don't imply the violation of the principle of causality and the negative propagation velocity. The negative group delay times do result from the reshaping [15] of the reflected wave packet, since each Fourier component undergoes the different phase shifts. We hope that this work may lead to novel applications in electronic devices, such as quantum mechanical delay line and high-speed electronic devices for two propagation directions [46].

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